Data-Based Mechanistic Modelling of Environmental, Ecological, Economic and Engineering Systems

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Abstract Mathematical modelling in the natural and engineering sciences is most often dominated by a philosophy of deterministic reductionism. Moreover, many of the 'simulation' models that emerge from this approach to modelling are very large and so difficult to identify, estimate (i.e. calibrate) and validate in rigorous statistical terms. In this situation, it seems sensible to consider alternative modelling strategies which overtly acknowledge these data-based modelling difficulties and address the very real problems of calibration and validation associated with the dynamic modelling of complex systems. This paper will outline a Data-Based Mechanistic (DBM) modelling philosophy and its associated methodological approaches to the analysis and modelling of data from environmental, ecological, economic and engineering systems. Here, the model structure is first identified and estimated, as objectively as possible, from the available data and the resulting model is only then interpreted in a scientifically acceptable and physically meaningful manner. This can be contrasted with the more conventional approach to model building in which the model structure is first defined on the basis of the scientist's perception of the system and then this model structure is used as the basis for estimation and validation exercises. The paper will discuss a number of practical examples including: extracting simplicity out of complexity in a large, nonlinear simulation model of the global carbon cycle used in climate change research; nonlinear rainfall-flow modelling; modelling the nonlinear feedback mechanism between blowflies and eggs in the Nicholson blowfly data; evaluating the possible causes of unemployment in the USA over the period 1948-1988; and the modelling and automatic control of the Harrier VSTOL Aircraft.

1. INTRODUCTION

As we look around us, we perceive complexity in all directions: environment, biological and ecological systems, national economics, and some of the more complex engineering systems - they all appear to be complicated assemblages of interacting processes, many of which are inherently nonlinear dynamic systems, often with considerable uncertainty about both their nature and their interconnections. It is not too surprising, therefore, that the mathematical models of such systems, as constructed by scientists, social scientists and engineers, are often similarly complex. What is surprising, however, is the apparently widespread belief[1] that such systems can be described very well, if not exactly, by *deterministic* mathematical equations, with little or no quantification of the associated uncertainty. Such *deterministic reductionism* leads inexorably to large, nonlinear simulation models which reflect the popular view that complex systems must be described by similarly complex models.

In the present paper, we present a different *Data-Based Mechanistic* (DBM) modelling philosophy which is almost the antithesis of deterministic reductionism. It is a philosophy built on our previous experience with the modelling of complex systems and emphasises the importance of parametrically efficient, low order, *dominant mode* models; as well as the stochastic methods and statistical analysis required for their identification and estimation. This approach to modelling is illustrated by five practical examples ranging from the characterisation of rainfall-flow dynamics for the purpose of flood forecasting, to the modelling and automatic control of the Harrier VSTOL aircraft. While these examples reject the concept of deterministic reductionism as the *major* approach to modelling, they recognise the value of simulation models and reductionist thinking in the overall modelling process. More importantly, however, they stress the importance of explicitly acknowledging the basic uncertainty that is essential to any characterisation of physical, chemical and biological processes, and argue for the greater utilisation of data-based statistical methods in the modelling of complex natural and man-made systems.

[1] Note, however, that econometricians, unlike many physical and natural scientists, cannot be accused of ignoring uncertainty: econometric models are inherently stochastic, albeit often based on methods which are overly based on the concepts of multivariate regression analysis.

2. DATA-BASED MECHANISTIC MODELLING

Previous publications (Young, 1978, 1983, 1992, 1993; Young and Minchin, 1991; Young and Lees, 1993; Young and Beven, 1994; Young *et al*, 1996) illustrate the evolution of the DBM philosophy and its methodological underpinning. This general approach is built on the assumption that the dynamic modelling of complex systems should involve two basic model types: speculative and normally quite complex simulation models which represent the current, state-of-the-art, scientific understanding of the system; and *Data-Based Mechanistic* (DBM) models obtained initially from the analysis of observational time-series but only considered credible if they can be interpreted in physically meaningful terms. The objective statistical derivation of these much simpler DBM models contrasts with the rather subjective formulation of the complex simulation models. However, the two, apparently quite different types of model are brought together in a rather novel phase of the analysis where the DBM methodology is used to simultaneously linearise and reduce the order of the complex simulation model, so exposing its 'dominant modes' of dynamic behaviour. This can then become a prelude to the final DBM modelling of the system from actual observational data sets obtained by monitoring the real system, either during its normal operation or, preferably, via planned experimentation.

The DBM philosophy and its associated methodological tools are discussed in the above references. However, the three major phases in the modelling strategy are as follows:-

1. In the initial phases of modelling, observational data may well be scarce, so any major modelling effort will have to be centred on simulation modelling,

normally based initially on deterministic concepts, such as dynamic mass and energy conservation. In the proposed approach, which is basically Bayesian in concept, these deterministic simulation equations are converted into a stochastic form by assuming that the associated parameters and inputs are inherently uncertain and can only be characterised in some suitable stochastic form, such as a probability distribution function (pdf) for the parameters and a time-series model for the inputs. The subsequent stochastic analysis uses *Monte Carlo Simulation* (MCS) in 3 ways: first, to explore the propagation of uncertainty in the resulting stochastic model; second, as a mechanism for *Generalised Sensitivity Analysis* (GSA) to identify the most important parameters which lead to a specified model behaviour; and third, the use of MCS in stochastic optimisation

2. The initial exploration of the simulation model in stochastic terms can reveal the relative importance of different parts of the model in explaining the dominant behavioural mechanisms. This understanding of the model is further enhanced by employing a novel method of combined statistical linearisation and model order reduction applied to time-series data obtained from planned experimentation not on the system itself but on the *simulation model* which, in effect, becomes a surrogate for the real system. This rather unusual *Dominant Mode Analysis* (DMA) is exploited in order to develop low-order, dominant mode approximations of the simulation model; approximations that are often able to explain its dynamic response characteristics to a remarkably accurate degree (e.g. coefficients of determination > 0.99). Conveniently, the statistical methods used for such linearisation and order reduction exercises are the same as those used for the DBM modelling from real time-series data that follows as the next stage in the modelling process

3. The DBM methods were developed primarily for modelling systems from normal observational time-series data obtained from monitoring exercises (or planned experimentation, if this is possible) carried out on the real system. In this stage of the proposed modelling approach, therefore, they are used to enhance the more speculative simulation modelling studies once experimental data become available. In this manner, the DBM models represent those *dominant modes* of the system behaviour that are clearly identifiable from the observational time-series data and, unlike the simulation models, the efficacy of the DBM models is heavily dependent on the quality of these data. This is, of course, both their strength and their weakness in practical terms.

Note that this approach emphasises the importance of observational data obtained from experiments or monitoring exercises on the real system and the need, wherever possible, to carefully plan such 'dynamic experiments' (see e.g. Goodwin and Payne, 1977) so that the dominant modes of system behaviour are clearly identifiable from these data. In this latter connection, the concept of scale in the measurements is very important for both data-based and simulation modelling. It is clearly not sensible to assume that a physically meaningful 'parameter' measured at the micro-scale (where 'micro' is, of course, relative to the size of the complete system) is the same as that which might be applicable at the aggregative, macro-level. DBM modelling, whether it is applied directly to real time-series data or used for developing simple, dominant mode representations of a large simulation model, yields mathematical relationships that are directly related *to the scale of the time-series measurements used in their derivation*. This needs to be acknowledged carefully when drawing deductions from the modelling results and interpreting the model in physically meaningful terms.

3. ILLUSTRATIVE PRACTICAL EXAMPLES

In practice, it is not always possible to exploit the whole DBM approach to modelling outlined in the previous section. Sometimes, as in the case of global carbon cycle models, time series data are scarce and it is necessary to concentrate on the first two, speculative modelling stages. On the other hand, in applications such as rainfall-flow modelling, data are often plentiful and flow forecasting applications, for instance, require only minimal order DBM models, thus largely negating the need for the first two stages. For this reason, each of the illustrative examples discussed in this section of the paper have been selected to show, in their different ways, how the analyst is able to select, from the overall DBM modelling strategy, only those methodological procedures that most suit the defined objectives of the example being considered.

3.1 Global Carbon Cycle (GCC) Models

At present, the main method used for research on global warming is the construction of computer-based mathematical models. Although such models can be of a very simple, empirical type, there seems to be a preference, amongst the scientific community studying climate change, for more complex and normally deterministic, dynamic simulation models. The most complex, costly and well known of these are the General Circulation Models (GCMs) whose mass and energy conservation equations, in the form of distributed parameter, partial differential equations, are so complex that they need to be solved by some form of numerical approximation in a super-computer. Rather less complicated but still of quite high dynamic order are the Global Carbon Cycle (GCC) models. Here, the movement of carbon in the global environment is described by a set of dynamic mass and/or energy conservation relationships in the form of lumped parameter, ordinary differential equations.

In this sub-section, we consider a typical deterministic, non-linear GCC model developed by Enting and Lassey (1993; hereinafter referred to as the EL model) on the basis of an original box-diffusion model suggested by Oeschger *et al* (1975). This 23rd order model is typical of the non-GCM models used by the Inter-governmental Panel on Climate Change (IPCC), the body set up to assess current scientific thinking on climate change and to advise on internationally co-ordinated policy in this area. The model is undoubtedly speculative but, despite the many uncertainties that underly its formulation, it is totally deterministic: the concept of uncertainty enters the analysis only when the model predictions into the next millennium are considered over a *range* of possible future deterministic scenarios; each of which, by definition, is speculative.

In this situation, Young *et al* (1996) concentrate on the first two stages in DBM modelling. The uncertainties in the model parameters and inputs required for the MCS analysis were obtained by reference to the latest literature on the subject and from information supplied by scientists working on global climate change. Table 1 compares the uncertainty measures associated with the EL model predictions of atmospheric CO_2 in the year 2100, as generated by the MCS analysis in the case of the IPCC scenario IS92a; and compares them with results obtained in well known deterministic scenario studies using large simulation models, including the EL model.

Table 1

Year	Uncertainty in CO_2 (ppmv)	Range (ppmv)	Modelling Source
2100	667 to 719	52	IPCC (1994)
	740 to 800	60	Wigley & Raper (1992)
	615 to 683	68	Enting & Lassey (1993)
	664 ± 144	288	original MCS results
	669 ± 48	96	MCS results with reduced uncertainty

Table 1: comparison of the uncertainty in future CO_2 levels due to IPCC scenario IS92a between three deterministic modelling exercises and the MCS methods used here.

The original MCS results show that the stochastic uncertainty is much larger than the range of variability found in the other studies. In order to avoid criticism that the selected uncertainty bounds on the parameters were unfair (see Parkinson and Young, 1997), the MCS analysis was repeated with various modification, including considerable reduction in the parametric and input uncertainties. The results of this revised simulation are also shown in Table 1 and, although the predictive uncertainty levels are much reduced, they remain 50% wider than the widest of the deterministic results. This is a significant amount: moreover, it would probably be larger still if the stochastic methodology could be applied to a complete set of global carbon cycle models, as in the IPCC's comparison of deterministic simulation studies.

To carry out the DMA, the full, non-linear EL model is initially set to an equilibrium condition with the (pre-industrial) atmospheric CO_2 concentration set at 275 ppmv. Then a small impulsive perturbation in the fossil fuel input is applied to the model and the resulting response is monitored over a period of 3000 years. The statistical identification and estimation analysis, using Simplified Refined Instrumental Variable (SRIV) methods of linear model identification and estimation (see Young *et al*, 1996, and the references therein), yields the 4th order, linear transfer function model, one interpretation of which is as a parallel connection of three, first order systems and an integrator (the latter is required because of the mass conservation assumption in the EL model). This simple dynamic model explains almost all (99.98%) of the simulated model response (i.e. the *Coefficient of Determination* (COD) based on the model output errors R_T^2=0.9998). More significantly, Figure 1 compares the response of this 4th order model with that of the full, 23rd order, non-linear model over the entire industrial period. As can be seen, the error is very small: never greater than 0.5ppmv, even though the model has moved 45 ppmv above the operating point at which it was estimated.

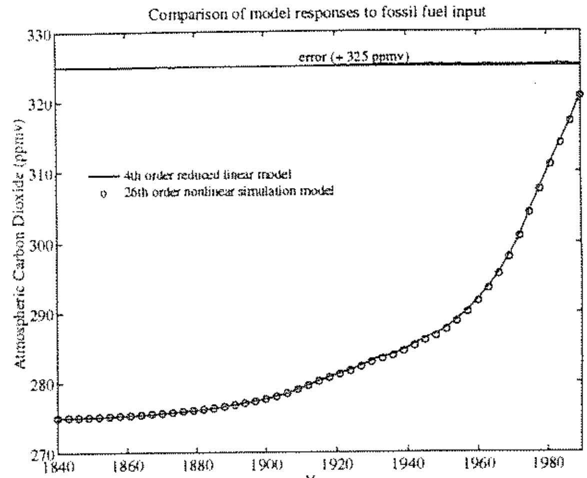


Fig.1 Combined model linearisation and reduction: comparison between the responses to fossil fuel input of the fourth order, linear model and the 23rd order nonlinear simulation model (error shown above (+325 ppmv)

These rather surprising results show the robustness of the low order, dominant mode model and suggest that the nonlinearities in the original model are hardly being excited by these fairly substantial perturbations over the industrial period. Moreover, further DMA results, at a range of different operating points, reveal that the objectively identified, 4th order model structure does not change at all; and the behaviour of the nonlinear model can be reproduced very closely, with only small variations in the parameter values. This is quite a dramatic result which demonstrates the dominance of a small number of modes of behaviour in the system, as defined by the eigenvalues of the reduced order, linear model at any defined operating point.

As regards the prediction of atmospheric CO_2, these results must call into question the need for such a complex representation of carbon balance in the EL model and suggests that a much simpler representation with fewer, or at least lower order, sub-systems (e.g. the EL model has 18 compartmental levels in the ocean sub-system alone) could have produced very similar results. Moreover, such a reduced order model would be more appropriate to the amount of observational data available in this example which, in itself, makes the assumption of a high order model rather questionable on statistical grounds. Indeed, the EL model can only be fitted to the available data with constraints applied to many of the parameters, a common indicator of severe over-parameterisation.

3.2 Rainfall-Flow Modelling

The nonlinear relationship between rainfall and flow data has been characterised mathematically in various ways: by complex, *physically-based simulation* models, such as the Stanford Watershed model (see e.g. Kraijenhoff and Moll, 1986), where the many spatially-distributed model parameters are obtained by rather *ad-hoc* methods, sometimes involving highly constrained optimisation; through the much more parsimonious *conceptual* lumped parameter models of the type suggested by Jakeman *et al* (1990) and Jakeman and Hornberger (1993), where the model structure and parameters have a conventional hydrologic interpretation but are obtained by more rigorous statistical estimation procedures from the rainfall-flow data; to the DBM models of Young *et al* (e.g. Young, 1993; Young and Beven, 1994).

A typical example of the DBM approach is the modelling of the data shown in fig.2.

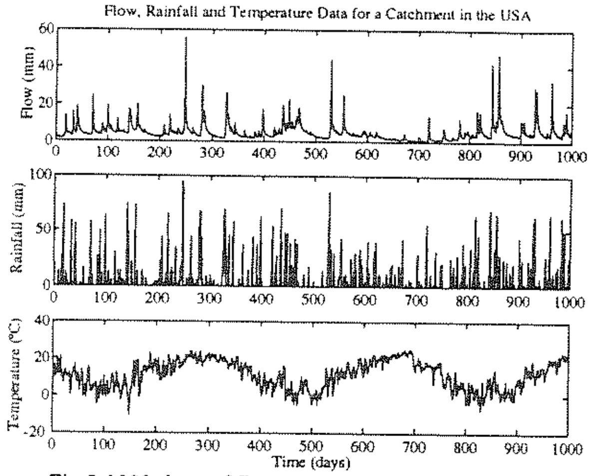


Fig.2 1000 days of flow (top), rainfall (middle) and temperature (bottom) data from a catchment in the USA.

On the basis of these data, the SRIV algorthm identifies and estimates the following, two input, TF model

$$y_t = 2.16 + \frac{0.4118 - 0.2819z^{-1} - 0.0430z^{-2}}{1 + 1.2330z^{-1} + 0.2952z^{-2}} u_{1t} - \frac{0.0041z^{-1}}{1 - 0.9691z^{-1}} u_{2t} + \xi_t \qquad (1a)$$

Here u_{2t} is the temperature variation about its mean value, which accounts for seasonal effects; the 2.16 is an apparently constant (over 1000 days) base flow effect; ξ_t is residual coloured noise; and u_{1t} is the effective rainfall input, defined as follows (see Young and Beven, 1994),

$$u_{1t} = \beta . r_t . y_t^{0.77} \qquad (1b)$$

where r_t is the measured rainfall and β is a normalisation coefficient. This model explains over 92% of the flow series with R_T^2=0.922, as shown in fig.3.

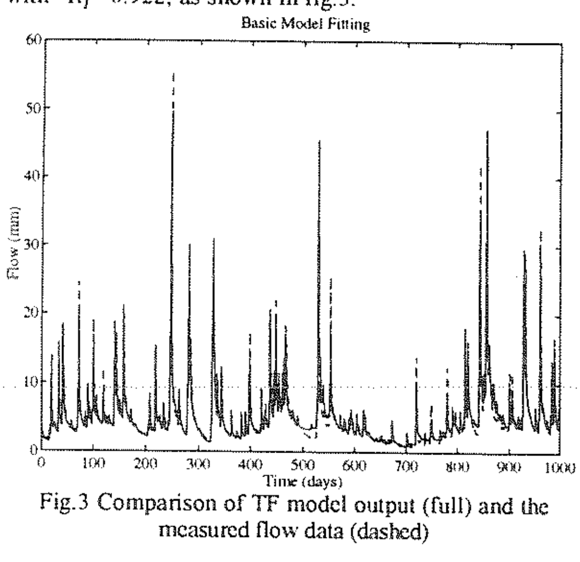
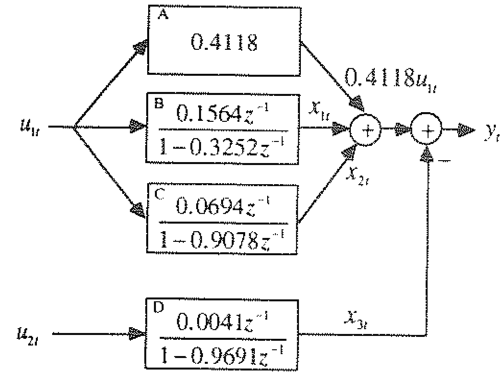


Fig.3 Comparison of TF model output (full) and the measured flow data (dashed)

It is interesting to look at this model in its decomposed parallel pathway form (the 'constant' base flow term has been removed for simplicity):

$$y_t = 0.4118u_{1t} + \frac{0.0694z^{-1}}{1-0.9078z^{-1}}u_{1t} + \frac{0.1564z^{-1}}{1-0.3252z^{-1}}u_{1t} - \frac{0.0041z^{-1}}{1-0.9691z^{-1}}u_{2t} \qquad (2)$$

or in block diagram terms:



We now see that *one interpretation* of the model is that the effective rainfall u_{1t} reaches the river and affects the flow via three pathways; and that the flow is also affected by the prevailing temperature variations which, presumably because of processes such as evapo-transpiration, reduce the flow in summer when temperature is greater than its mean value (see the negative sign on the y_t summation). The details of the individual first order TF's in the decomposition (now including the constant base flow effect) are:

Instantaneous TF (A)

Root	SSG	TC	%flow
0	0.4118	0	29.4980

Fast Flow TF (B)

Root	SSG	TC	%flow
0.3252	0.2318	0.8902	16.6092

Slow Flow TF (C)

Root	SSG	TC	%flow
0.9078	0.7523	10.3370	53.8928

Slow Temperature (Seasonal) Effect (D)

Root	SSG	TC
0.9691	0.1327	31.9

Constant Base flow

2.16 ($\forall t$)

where SSG and TC denote, respectively, the associated steady state gain and time constant (residence time) of the individual first order TF's. The outputs of the various parallel pathways, together with the temperature effect, are shown in fig.4.

To summarise, the TF modelling analysis in this case provides a 'black box' model but, by decomposing this model, we obtain a mechanistic interpretation that makes physical sense. In particular, it suggests that the river flow is composed of 5 components: a very rapid instantaneous (i.e. within one day) effect; a 'fast flow' component with residence time 0.89 days; a 'slow flow' component with residence time 10.34 days; and a very slow, 'base flow' component consisting of a 'constant flow' (2.16 mm: constant over the 1000 days of data); and a temperature dependent component (the temperature variations about the mean passed through a TF with time constant 31.9 days), which appears to account for long term temperature dependent effects, such as those arising from evapo-transpiration processes. Moreover, the TF's associated with 'fast' and 'slow' flow parallel pathways can be interpreted as a dynamic mass balance or storage equations (see Young and Beven, 1994).

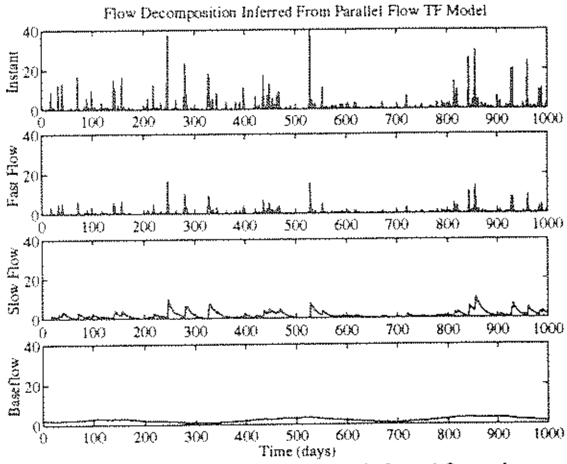


Fig.4 Decomposition of the flow, as inferred from the parallel pathway model (note all flows are to the same scale)

Rainfall-flow models such as (1) are not only useful for gaining an improved understanding of the nonlinear catchment dynamics and allowing the hydrologist to quantitatively decompose the flow into surface and 'baseflow' components (e.g. Jakeman *et al*, 1990), they also provide a valuable tool in the design of flood warning and forecasting systems (e.g. Lees *et al*, 1994; Young *et al*, 1997a).

3.3 The Nicholson Blowfly Data

Over a number of years, Nicholson (see e.g. 1954) collected data from a series of experiments involving captive colonies of the Australian sheep blowfly, *Lucilia cuprina*. These data, which are records of the variations in the numbers of eggs, larvae and adult blowflies over extensive time periods, have been analysed by numerous ecologists and mathematicians in the intervening period of time. In contrast to these earlier studies, which tend to concentrate mainly on the nonlinear behaviour of the blowflies, the DBM approach used here considers the complete egg-blowfly system and evolves a nonlinear model relating the eggs and blowflies within a closed, nonlinear, feedback loop.

The best known of Nicholson's data sets[2] is shown in fig.5. These data were obtained with the blowflies subjected to a limited food supply of 0.5 g/day ground liver and fig.5 suggests classic, nonlinear, limit cycle behaviour. The DBM analysis begins with the identification and estimation of the 'forward path' dynamics between the eggs and blowflies, where a linear relationship seems most likely. SRIV identification and estimation confirms a first order linear model characterised by a 15 day pure time delay which accounts for egg-larvae development time. In contrast, however, the analysis suggests that the feedback dynamics, defining the egg-laying behaviour of the blowflies, is clearly nonlinear.

[2] the original data were lost by McNeil (1996) and the present data were digitised from Nicholsons's original paper. An earlier paper (Young and Chotai, 1997) used a different digitised set of data which was later found to have some temporal distortion. The present results supersede these earlier results, although there is little qualitative difference.

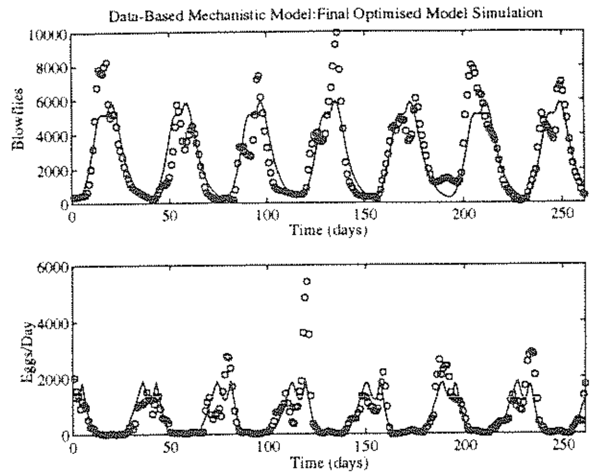


Fig.5 Comparison of the DBM nonlinear model output (full line) and the digitised Nicholson data (circles): the blowfly variations are shown above and the egg laying rate variations below.

The DBM analysis outlined here is taken from Young (1997). The initial data-based nonlinear modelling method used to investigate the blowfly-egg dynamics is a non-parametric estimation procedure based on recursive Fixed Interval Smoothing (FIS), as discussed in Young (1993) and Young and Beven (1994). In the top graph of fig.6, the resulting estimate of the nonlinearity is plotted as circles with 3x(standard error) band shown dashed: it suggest that at low blowfly populations the egg production is approximately proportional to the population; while at higher populations, when the food supply per blowfly is getting progressively smaller, there is a gradual decrease in egg production from its maximum level until very few eggs are laid each day for blowfly populations greater than about 4000.

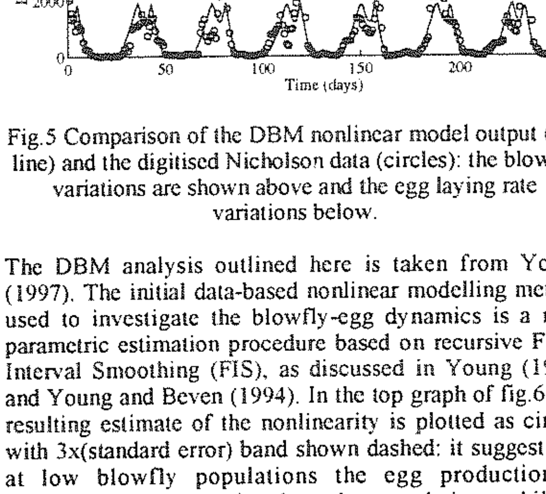


Fig.6 Non-parametric and parametric estimates of the feedback nonlinearityin the blowfly model.

The middle and lower graphs of fig.6 show a particular parameterisation of the nonlinearity: this is plotted directly in the middle graph; and in terms of the food/blowfly and eggs/day/blowfly, in the lower graph. This parameterised nonlinearity, which consists of a linear segment for blowfly populations y_t<518 (see below) and an exponential decrease thereafter, is also plotted as a dash-dot line in the upper graph, where it can be compared directly with the initial non-parametric estimate.

The above results were obtained by fitting the following nonlinear model to the egg-blowfly data using numerical optimisation based on a simple least squares cost function:

$$
\begin{aligned}
y_t &= a y_{t-1} + b u_{t-15} \\
u_t &= \alpha y_t & &: \text{if } y_t \leq y_o \qquad (3) \\
u_t &= \alpha . y_o . \exp\{-(y_t - y_o)/(f.N)\} & &: \text{if } y_t > y_o
\end{aligned}
$$

Here f is the food supply per day (0.5 g in this case); f_o is the optimum food supply per blowfly, i.e. the per capita food supply at an optimum blowfly population $y_o = f/f_o$, which yields the maximum egg production rate. The estimated parameters are as follows:

$$a = 0.828(0.002);\ b = 0.719(0.032);\ \alpha = 3.74(0.36);$$
$$f_o = 9.6\text{x}10^{-4}(7.7\text{x}10^{-5});\ y_o = 518(16);\ N = 1885(128)$$

where the figures in parentheses are the estimated standard errors.

The deterministic, limit cycle output of this optimised model is shown as the full line in fig.5, where it can be compared with the Nicholson experimental data. As can be seen, the model explains the data well with R_T^2=0.77 for the blowflies and R_T^2=0.58 for the eggs. In addition, it can be independently validated since Nicholson states that, "*The culture ... was supplied with 0.5 g ground liver per day and the average density was found to be 2520. In another culture in which all the conditions were precisely the same, except that only 0.1 g of liver was provided per day for the adults, the average density of adults was 527*". In the case of the above model, the average density in the case of 0.5 g liver per day is 2589, and when this is reduced to 0.1 g per day, the average density reduces to 527. Clearly these are in remarkable agreement with Nicholson's results.

Of course, equation (3) is not the only parameterisation that could be used (see Young, 1997) but it has the virtue of a clear ecological interpretation, which is most important in DBM modelling. For instance, the parameters a and b define the average survival rates of the blowflies (82.8%) and eggs-larvae (71.9%), respectively; y_o (518) and f_o (9.6x10^{-4}) are, in terms of egg production, the 'optimal' blowfly population and food supply rate per blowfly, respectively; and the pure time delay (15 days) is the time taken for the egg-larvae stage in the blowfly life cycle.

3.4 Macro-Economic Relativity: Unemployment and Investment in the USA 1945-1988

Macro-economic data are often non-stationary, in the sense that they are characterised by long term stochastic trends which may have common characteristics. The major current approach to handling such series is the concept of *co-integration* (Engle and Granger, 1987). A recent DBM macro-economic modelling study (Young and Pedregal, 1997a,b), again exploiting non-parametric estimation based on recursive FIS estimation (as used in the previous blowfly example), has suggested an alternative approach, in which the non-stationary series are replaced by suitable relativistic measures, so inducing near-stationarity and removing the need for the inclusion of stochastic trends in the model.

The study is concerned with modelling and forecasting the percentage unemployment rate y_t (itself a relativistic measure) in the USA over the period 1945-1988, based on the changes in relative private investment RPI_t and relative Government spending (i.e. public investment) RGI_t, where both are measured relative to the Gross National Product (GNP). The three series, based on quarterly measures, are shown in fig.7. It is clear from these plots that there has been a reduction of total relative investment $RI_t = RPI_t + RGI_t$, since around 1970, that is due primarily to the decline of public rather than private investment relative to GNP: in particular, standard statistical tests show unambiguously that the mean level of RGI_t has declined significantly from a roughly constant level of 24.48±0.69% of GNP in the period 1955-1969, to 20.07±0.72% of GNP, in the period 1973-1988; meanwhile relative private investment RPI_t, whilst very volatile in the short term, has remained at a roughly constant mean level (16.68±1.45.% of GNP) over the whole period.

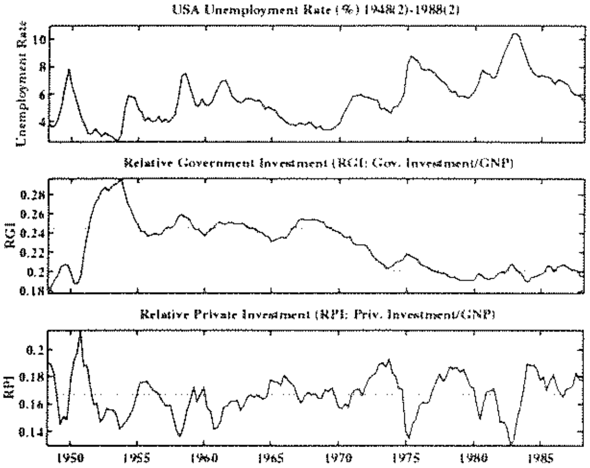


Fig.7 Unemployment Rate y_t, Relative Government Investment RGI_t, and Relative Private Investment RPI_t.

Refined Instrumental Variable (RIV) identification and estimation (see Young and Jakeman, 1979; Young, 1984) applied to the data in fig.7 yields the following TF model,

$$y_t = c + \frac{b_1}{1+a_1 L} RGI_t + \frac{b_2}{1+a_1 L} RPI_t + \frac{1}{1+c_1 L + c_2 L^2} e_t$$

where e_t is zero mean, white noise with variance σ^2. The full estimation results are reported in Table 2, in which R_T^2 is the COD based on the model outputs; while R^2 is the COD based on the one-quarter-ahead prediction errors.

Table 2

Parameter	Estimate	SE	T statistic
c	10.178	0.901	11.29
a_1	-0.777	0.028	28.12
b_1	-15.081	1.734	8.68
b_2	-32.981	2.758	11.96
c_1	-1.016	0.077	13.14
c_2	0.246	0.077	3.18

σ^2= 0.1015 R_T^2=0.894 R^2=0.965

Steady State Gains: G_1=-67.64; G_2=-147.92;

Time constant, T_c=3.96 quarters

Table 2: Estimated TF model between US unemployment rate and the ratios to GNP of Private and Public Investment. The noise is modelled as an AR(2) model.

Clearly, the model explains the data well and also produces excellent short term predictions. In addition, it performs well in long term forecasting terms: fig. 8, for example, shows the ten-year-ahead unemployment rate forecasts produced on the basis of various forecasts of RPI_t and RGI_t. It will be seen that the best *true* forecast is produced using *Dynamic Harmonic Regression* (DHR) for these input variables: this is another methodological tool (see e.g. Young, 1988; Young *et al*, 1989; Young *et al*, 1997b) that often proves very useful in DBM modelling studies, where it can provide an explanation for non-stationary periodic and quasi-periodic series and effects.

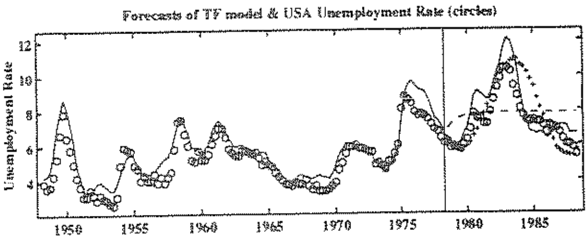


Fig.8 Unemployment Rate in USA (circles) compared with forecasts when: (a) the actual values of the explanatory variables are used (full); (b) they are forecast using DHR models (crosses); (c) they are assumed to remain constant at their local mean level before the forecast origin (dashed).

Although it leads to an excellent explanation of the data and produces good forecasts, the analysis presented in this paper does not, of course, *prove* that the modelled relationships are causal: it is extremely difficult, if not impossible, to prove that causative mechanisms exist between economic variables because the variables are simply observed (measured) passively during the normal operation of the system, and planned experiments, which could remove the ambiguity, are not possible. But, at the same time and for the same reasons, causality can rarely be *disproved* and so it is very important to acknowledge the *possibility*, as revealed by our analysis, of a causal connection and to take this into account in risk-sensitive economic planning, particularly if the data-based conjecture conforms with some important aspect of economic theory. This linking of the model with theory is the important last stage in DBM modelling: no matter how well the time series model explains and forecasts the data, it cannot really be considered fully credible in a truly scientific sense unless it can be interpreted in meaningful physical (here macro-economic) terms. In the present case, therefore, it is fortunate that the model appears to conform well with certain aspects of Keynesian economic theory, as discussed in Young and Pedregal (1997b).

3.5 Autostabilisation of the Harrier VSTOL Aircraft

The final example is concerned with the design of an advanced automatic control system for the Harrier Vertical and Short Take Off and Landing (VSTOL) aircraft in its most difficult transitional mode between hovering and normal flight (Chotai *et al*, 1997). This design study is based on a realistic, high order differential equation model in *Simulink*, characterised by nonlinear actuator dynamics, with both amplitude and rate limits, as well as sensor dynamics which include additional pure time delays. The design objectives are to obtain a fast reacting, dynamically decoupled, closed loop system which is stable within a relatively wide range of command input values and functions well even when the amplitude and rate limits are activated.

In this example, where the design is based on the high order nonlinear simulation model rather than experimental data from flight tests, the identification and estimation analysis is used for simultaneous model linearisation and reduction based on simulation experiments (as discussed in section 3.1). Once again, SRIV identification and estimation (this time using the delta operator form of the algorithms: see Young *et al.* 1991) consistently provides well defined, low (7th) order, linear 'dominant mode' control models which explain almost all of the, nominally nonlinear, simulation model data for a fairly wide range of input amplitudes (i.e. coefficients of determination $R_T^2 \cong 1$).

The control system design methods utilised in the study are based on techniques developed in the Systems and Control Group of CRES over a number of years (Young *et al.* 1987, 1991, 1997c; Chotai *et al.* 1997). This involves a *Non-Minimum State Space* (NMSS) approach, which results in a multivariable *Proportional-Integral-Plus* (PIP) control system that provides a logical, optimal development of conventional PID controllers. In this case, the design requirements are very demanding and a multi-objective optimisation method has been utilised to extract maximum performance from the PIP controller (see Chotai *et al*, 1997).

Fig. 9 shows the closed loop response to a unit step pitch attitude command input. The top graph is the pitch channel output response, along with the other two, dynamically decoupled outputs (forward and vertical velocity) plotted to the same scale. The three graphs below this plot show the control input values, all of which are active in ensuring the decoupled response. Note that these results were obtained using the *full non-linear simulation*, rather than the linear, reduced order model used in the multi-objective optimisation stage of the design analysis.

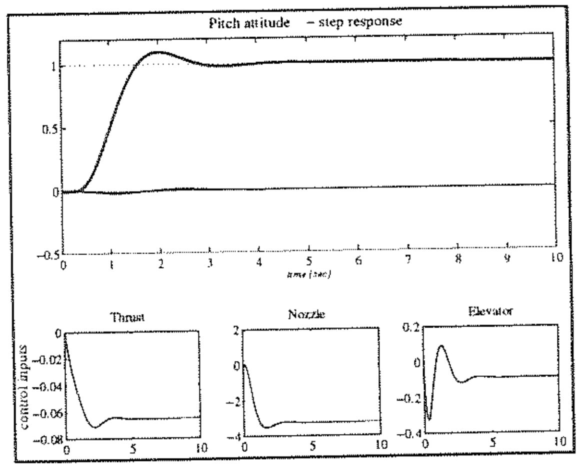


Figure 9. Response of PIP controlled aircraft to a command step input on pitch attitude.

Clearly, the design objectives, i.e. relatively fast and decoupled responses, have been achieved. The command step size is the same as that used for optimisation, and the actuator amplitude and rate limits are not exceeded. A number of additional closed loop nonlinear simulation tests, including step and random (gust) load disturbances, as well as changes in the parameters of the rigid body part of the non-linear model, have shown that the closed loop behaviour is robust to such effects. If larger command input signals are applied, the input signals can exceed the amplitude and rate limits but the PIP system continues to perform well, with only a small decrease in performance under these conditions.

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